

# State-Space Design Method for Motor Position Control

## Overview

This tutorial shows how to use the state-space design method for DC motor position control. This tutorial uses LabVIEW 8.2 with LabVIEW Control Design Toolkit.

These tutorials are based on the [Control Tutorials](#) developed by [Professor Dawn Tilbury](#) of the Mechanical Engineering department at the University of Michigan and [Professor Bill Messner](#) of the Department of Mechanical Engineering at Carnegie Mellon University and were developed with their permission.

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From the main problem, the dynamic equations in state-space form are the following:

$$\frac{d}{dt} \begin{bmatrix} \theta \\ \dot{\theta} \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{K}{J} \\ 0 & -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} V$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ i \end{bmatrix}$$

For the original problem setup and the derivation of the above equations, please refer to the [Modeling DC Motor Position](#) page.

With a 1 rad reference added to the system, the design criteria are:

- Settling time less than 0.04 seconds
- Overshoot less than 16%
- Zero steady-state error to a step input
- Zero steady-state error to a step disturbance

Open the MathScript Window (Tools &#62;&#62; MathScript Window) and click on the Script tab to view the Script Editor. Type the following commands in the Script Editor:

J = 3.2284E-6;

b = 3.5077E-6;

K = 0.0274;

R = 4;

$L = 2.75E-6;$

$A = [0 \ 1 \ 0$

$0 \ -b/J \ K/J$

$0 \ -K/L \ -R/L];$

$B = [0 \ ; \ 0 \ ; \ 1/L];$

$C = [1 \ 0 \ 0];$

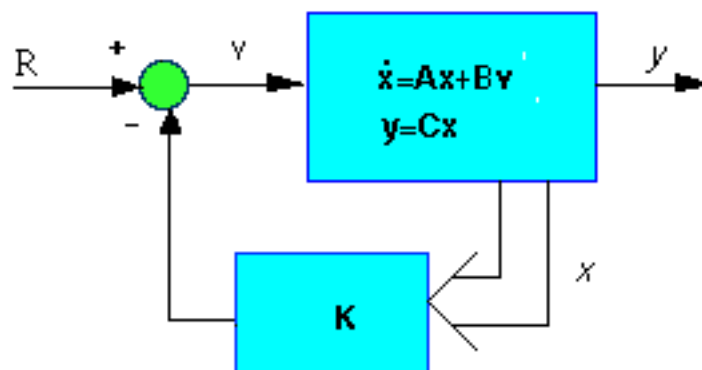
$D = [0];$

$\text{sys}=\text{ss}(A,B,C,D);$

## Designing the Full-State Feedback Controller

Since all of the state variables in our problem are very easy to measure (simply add an ammeter for current, a tachometer for speed, and a potentiometer for position), we can design a full-state feedback controller for the system without worrying about having to add an observer.

The schematic for a full-state feedback system is:



Recall that the characteristic polynomial for this closed-loop system is the determinant of  $(sI - (A - BKc))$  where  $s$  is the Laplace variable. Since the matrices  $A$  and  $B*Kc$  are both  $3 \times 3$  matrices, there should be 3 poles for the system. By designing a full-state feedback controller, we can move these three poles anywhere we want them.

We shall first try to place them at  $-100 + 100i$  and  $-100 - 100i$  (note that this corresponds to a  $\zeta = 0.5$ , which gives 0.16% overshoot, and a  $\sigma = 100$ , which leads to a .04 sec settling time).

Once we come up with the poles we want, LabVIEW will find the controller matrix,  $Kc$ , for us. Simply add the following code to the end of your script:

$p1 = -100 + 100i;$

```
p2 = -100-100i;
```

```
p3 = -200;
```

```
Kc = place(A,B,[p1,p2,p3]);
```

Now look at the schematic above again. We see that after adding the K matrix into the system, the state-space equations become:

$$\dot{\mathbf{X}} = (\mathbf{A} - \mathbf{BK})\mathbf{X} + \mathbf{Bu}$$

$$\mathbf{Y} = \mathbf{CX}$$

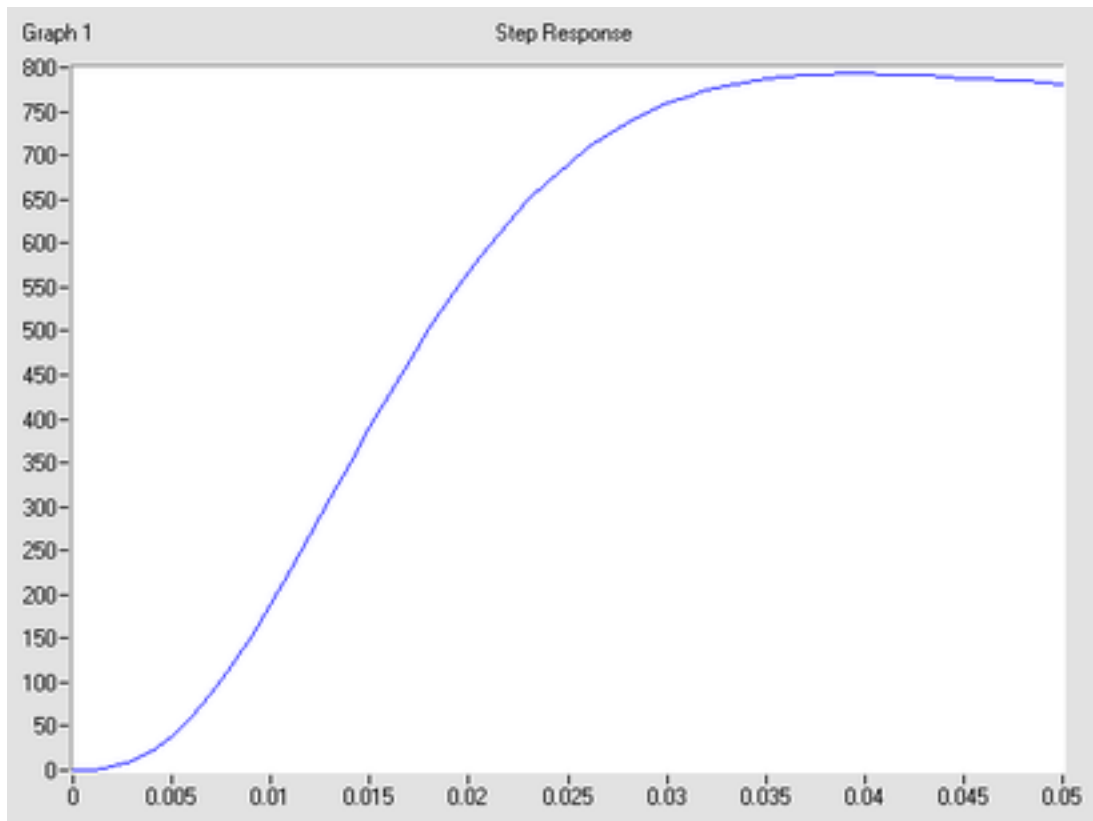
To see the closed-loop response, add the following lines to your existing code:

```
t = 0:0.001:.05;
```

```
sys_cl=ss(A-B*Kc,B,C,D);
```

```
step(sys_cl,t)
```

When you run the script, you should see the following plot:



**Figure 1: Closed-Loop Step Response with Full State Feedback**

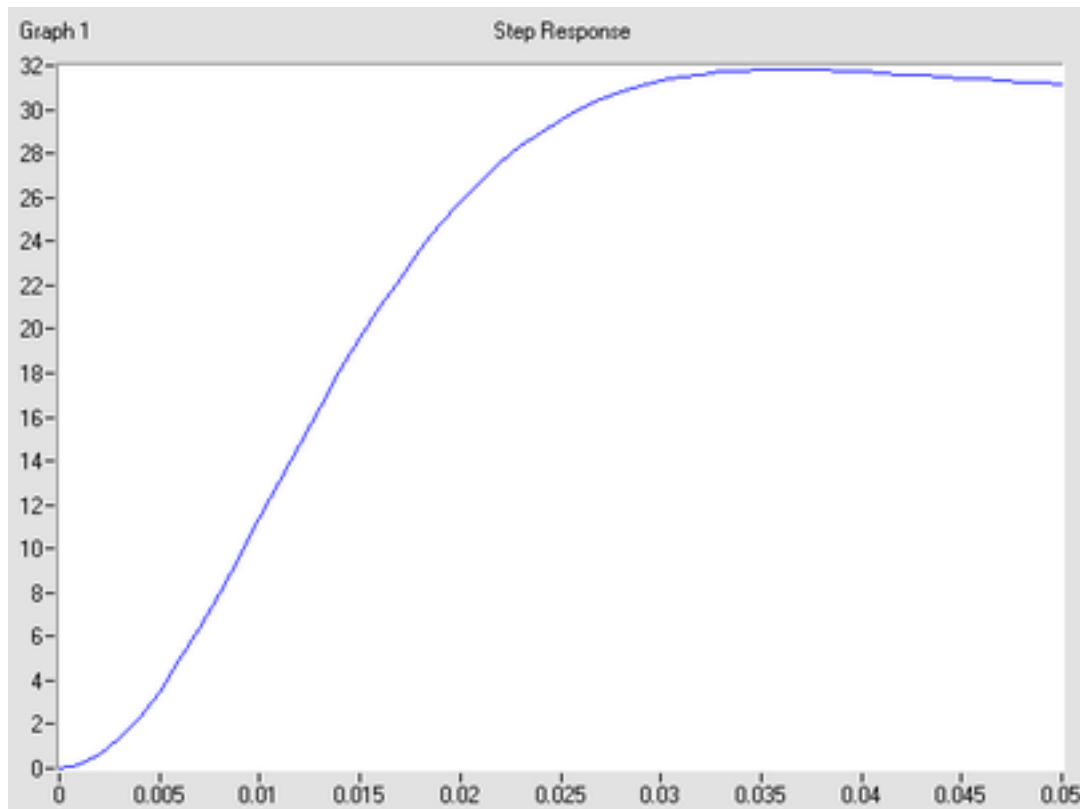
## Disturbance Response

In order to get the disturbance response, we must provide the proper input to the system. Physically, a disturbance is a torque which acts on the inertia of the motor. A torque acts as an additive term in the second state equation (which gets divided by  $J$ , as do all the other terms in this equation).

We can simulate this simply by modifying our closed loop input matrix,  $B$ , to have a  $1/J$  in the second row. Add the following code to your existing script and re-run it.

```
dist_cl=ss(A-B*Kc,[0;1/J;0],C,D);  
step(dist_cl,t)
```

You should see the following plot when you run the script:



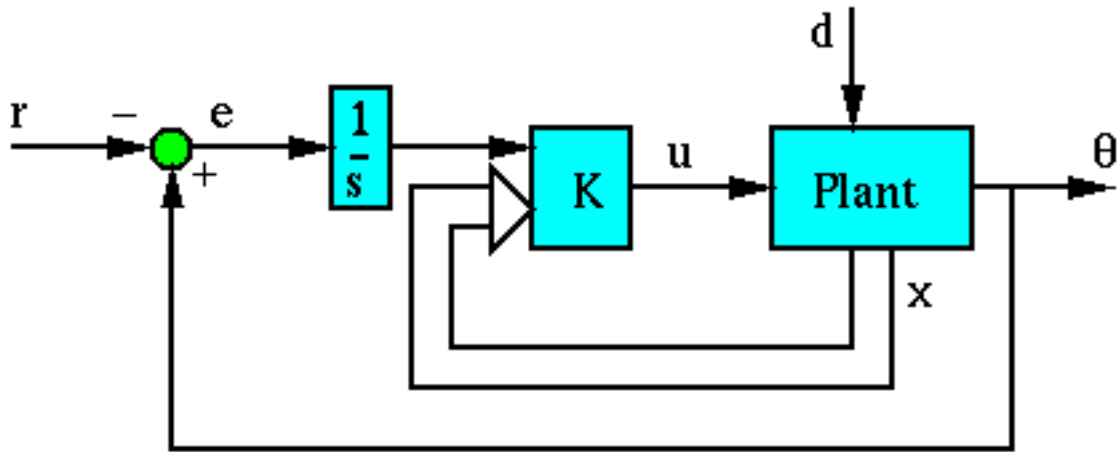
**Figure 2: Disturbance Response with Full State Feedback**

This is not a zero steady-state error to a disturbance, and we will have to compensate for this.

## Adding Integral Action

We know that if we put an extra integrator in series with the plant it can remove steady-state error to an input. If the integrator comes before the injection of the disturbance, it will cancel the disturbance in steady state.

This changes our control structure so it now resembles the following:



We can model the integrator by augmenting our state equations with an extra state which is the integral of the output. This adds an extra equation which states that the derivative of the integral of theta is theta. This equation will be placed at the top of our matrices.

The input, r, now enters the system before the integrator, so it appears in the newly added top equation. The output of the system remains the same.

$$\frac{d}{dt} \begin{bmatrix} \int \theta \\ \theta \\ \dot{\theta} \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{b}{J} & \frac{K}{J} \\ 0 & 0 & -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \int \theta \\ \theta \\ \dot{\theta} \\ i \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \int \theta \\ \theta \\ \dot{\theta} \\ i \end{bmatrix}$$

These equations represent the dynamics of the system before the loop is closed. We will refer to the matrices in this equation as Aa, Ba, Ca, and Da. We will refer to the state vector of the augmented system as xa. Note that the reference, r, does not affect the states (except the integrator state) or the output of the plant - this is expected, since there is no path from the reference to the plant input, u, without implementing the feedback matrix, Kc.

In order to find the closed loop equations, we have to look at how the input, u, affects the plant. In this case, it is exactly the same as in the unaugmented equations. Therefore, there is a vector, call it Bau, which replaces Ba when we are treating u as the input. This is just our old B vector with an extra zero added as the first row. Since  $u = Kc * x_a$  is the input to the plant for the closed loop, but r is the input to the closed loop system, the closed loop equations will depend on both Bau and Ba.

The closed loop equations will become:

$$\dot{x}_A = (A_A - B_{A11}K) x_A + B_A r$$

$$\theta = C_A x_A$$

Now, the integral of the output will be fed back, and will be used by the controller to remove steady state error to a disturbance. We can now redesign our controller. Since we need to place one closed-loop pole for each pole in the plant, we will place another pole at -300, which will be faster than the rest of the poles. Since the closed-loop system matrix depends on  $B_{Au}$ , we will use  $B_{Au}$  in the place command rather than  $B_A$ .

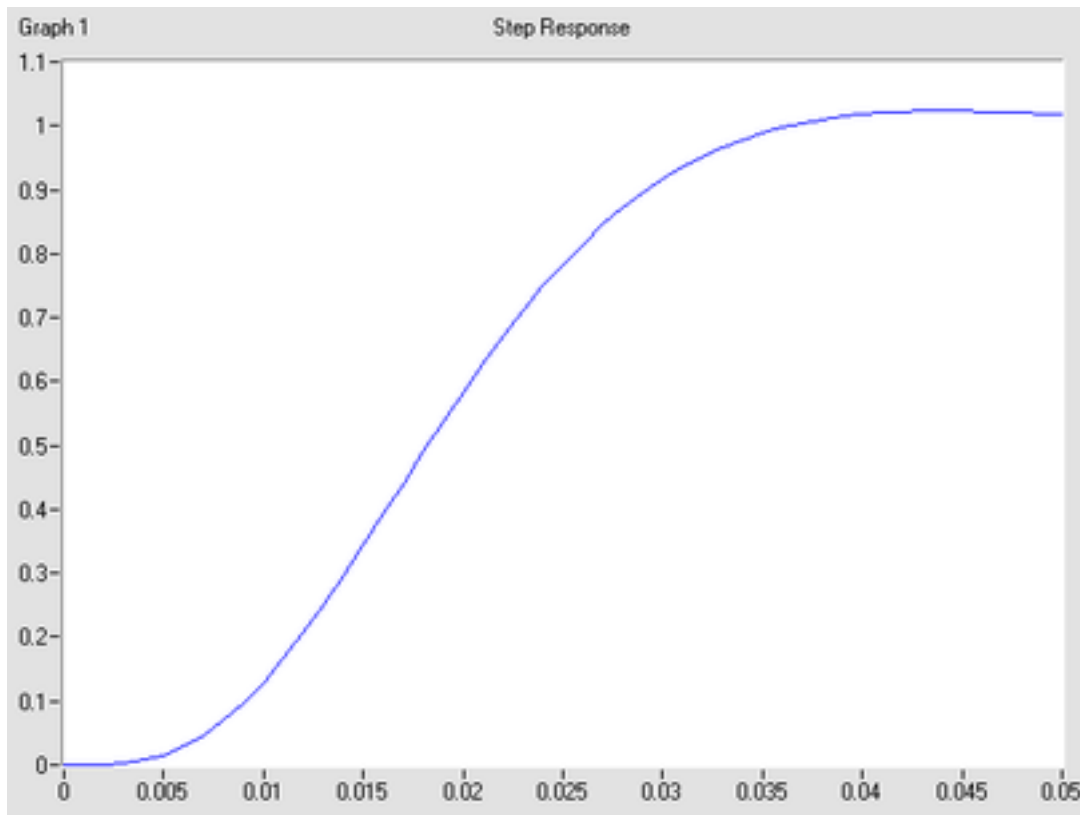
Add the following code to your existing script:

```
Aa = [0 1 0 0
      0 0 1 0
      0 0 -b/J K/J
      0 0 -K/L -R/L];
Ba = [-1 ; 0 ; 0 ; 0];
Bau = [0 ; 0 ; 0 ; 1/L ];
Ca = [0 1 0 0];
Da = [0];

p4 = -300;
Kc = place(Aa,Bau,[p1,p2,p3,p4]);

t = 0:0.001:.05;
sys_cl = ss(Aa-Bau*Kc,Ba,Ca,Da);
step(sys_cl,t)
```

Run your script, and you will get the following output:

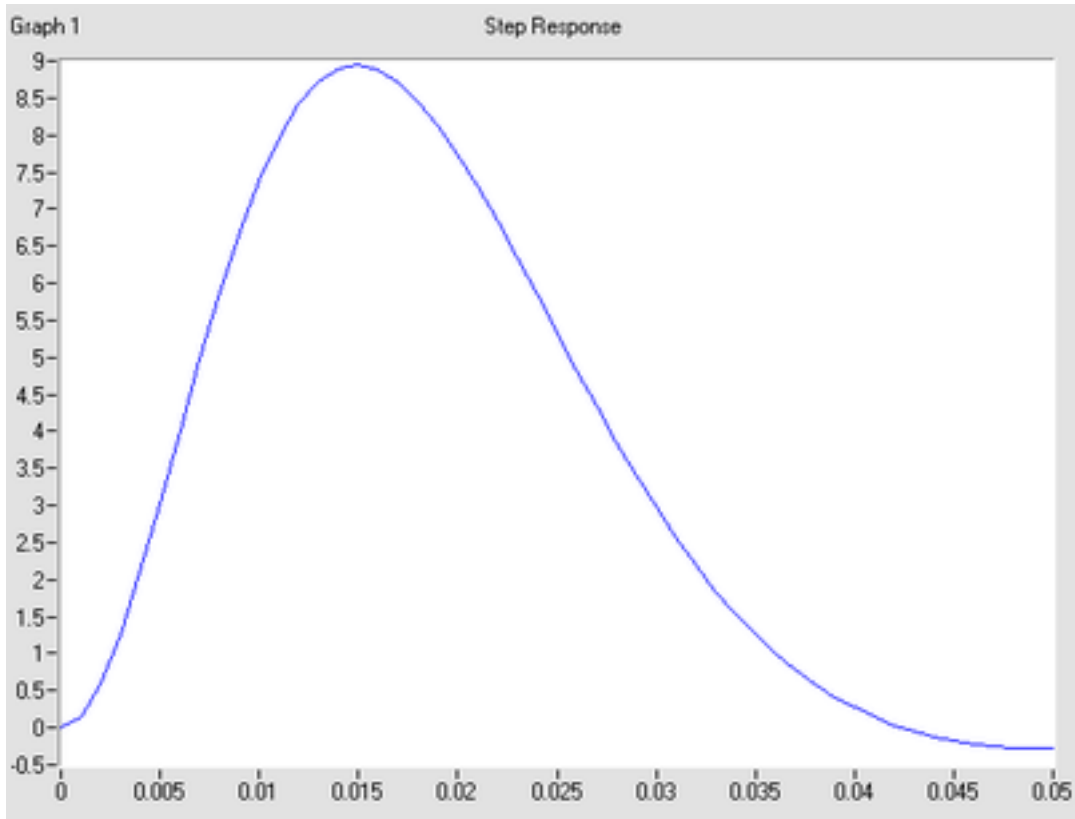


**Figure 3: Step Response - Full State Feedback with Integrator**

To look at the disturbance response, we apply a similar B matrix as we did previously when simulating the disturbance response.

```
dist_cl = ss(Aa-Bau*Kc,[0 ; 0 ; 1/J ; 0],Ca,Da);
```

```
step(dist_cl,t)
```



**Figure 4: Disturbance Response - Full State Feedback with Integrator**

We can see that all of the design specifications have been met by this controller.

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